2018-2019 School Year: Due first day of class Second Semester

Massey Hill Classical High School

AB Calculus AP Prerequisite Packet

To: AP Calculus AB Students and Parents
From: Mr. Joseph B. Miller, AP Calculus Instructor

The AP Course: AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. The course is taught in one semester consisting of 90 -minute classes.

The Prerequisite Packet: Students need a strong foundation and willingness to be ready for the rigorous work required throughout the term. Completing the prerequisite packet should prepare you for the material to be taught in the course. This packet consists of material studied during Math I through Math III and Pre- Calculus Honors. Students should anticipate working approximately 10 hours to complete it properly.

The packet will be collected on the **FIRST day of class** and your grade will be determined on neatness, completeness of solutions, and accuracy. In preparation for the AP test students need to begin showing all work with logical steps. Do not list only an answer. Work neatly and in an organized fashion.

Calculators: Students enrolled in AP Calculus AB will be using a graphing calculator throughout the course. A graphing calculator is required on the AP test. We will be using a TI 84 Plus and TI 89 in class;, I recommend a TI 84 Plus Silver Edition and/or the TI-89. If you have a TI-83 calculator some of our needed programs can be downloaded onto your calculator.

Since the AP test is offered only in May, students taking only the AB course should study for the test independently and attend any after school review sessions or practice test sessions that are offered. It is the student's responsibility to find out when these sessions will be held.

I anticipate a motivating and challenging year. Calculus is a stimulating and exciting field of mathematics and I look forward to sharing the excitement with you. I will be there to help and support you.

SKILLS NEEDED FOR CALCULUS

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
F. Simultaneous equations

II. Graphing and Functions

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
B. Conic Sections (circle, parabola, ellipse, and hyperbola)
*C. Functions (definition, notation, domain, range, inverse, composition)
*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric. piece-wise, inverse functions)
E. Tests for symmetry: odd, even

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

* *IV. Logarithmic and Exponential Functions*

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

* *V. Trigonometry*

**A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

VI. Limits

A. Concept of a limit
B. Find limits as x approaches a number and as x approaches ∞

* A solid working foundation in these areas is very important.

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AP Calculus AB – Summer Work – 1.1 Linear Equations

Find the slope from the following points, graphs, and linear equations.

1. $p_1 = (2,1)$ and $p_2 = (1,4)$

Slope: _____

2. $p_1 = (-3,-2)$ and $p_2 = (0,7)$

Slope: _____

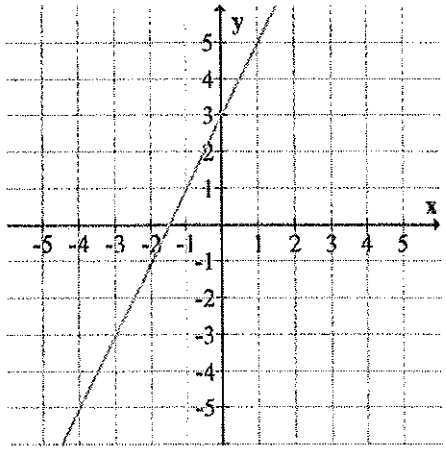
3. $p_1 = (2,2)$ and $p_2 = (4,2)$

Slope: _____

4. $p_1 = (3,-2)$ and $p_2 = (3,7)$

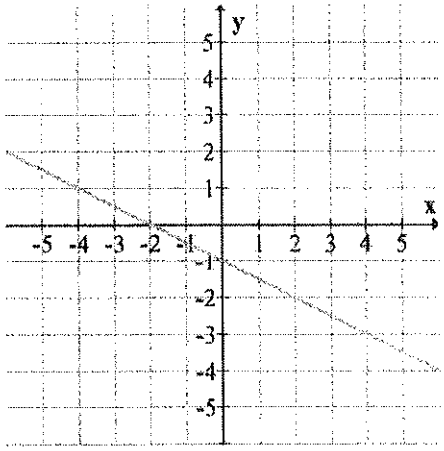
Slope: _____

5.



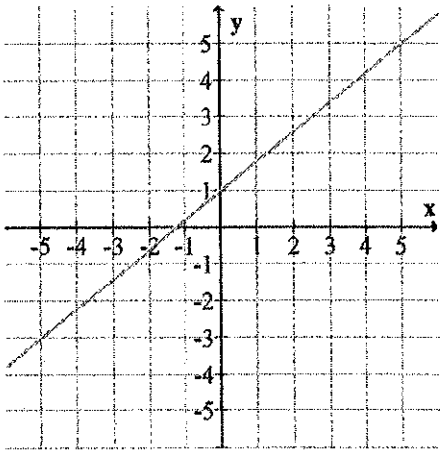
Slope: _____

6.



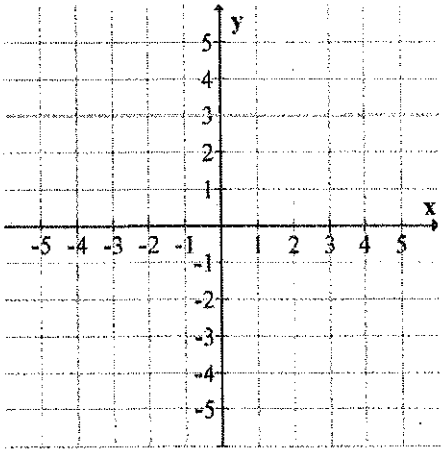
Slope: _____

7.



Slope: _____

8.



Slope: _____

9. $y = -2x - 3$

Slope: _____

10. $y - 5 = \frac{2}{3}(x+1)$

Slope: _____

11. $3x+4y=8$

Slope: _____

12. $x=-5$

Slope: _____

Write the equation of the line in all three forms given a slope and a point.

13. $m=\frac{2}{3}$ and $p=(3,5)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

14. $m=-\frac{4}{5}$ and $p=(1,2)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

Write the equation of the line in all three forms given a slope and a y-intercept.

15. $m=-3$ and $y-\text{intercept}=5$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

16. $m=-\frac{3}{2}$ and $y-\text{intercept}=-3$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

Write the equation of the line in all three forms given two points.

17. $p_1=(2,2)$ and $p_2=(4,2)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

18. $p_1=(3,-2)$ and $p_2=(3,7)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

19. $p_1=(2,1)$ and $p_2=(1,4)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

20. $p_1=(-3,-2)$ and $p_2=(0,7)$

Point-Slope: _____

Slope-Intercept: _____

Standard Form: _____

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AP Calculus AB – Summer Work – 1.2 Functions and Graphs

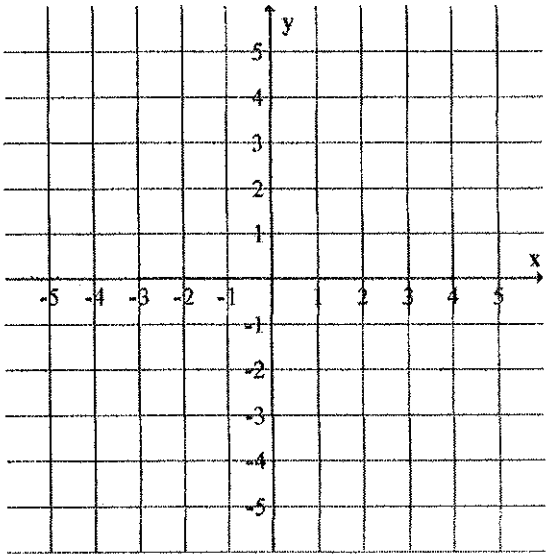
For the following functions find the a) domain, b) range, c) graph, and d) any symmetries.

1. $y = 4 - x^2$

a) _____

b) _____

c)



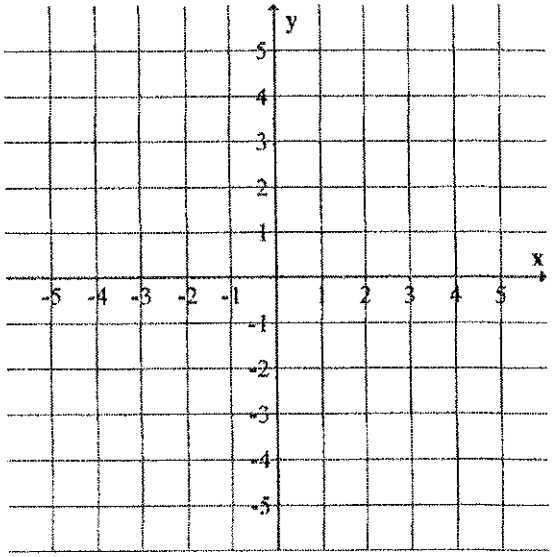
d) _____

2. $y = 2 + \sqrt{x-1}$

a) _____

b) _____

c)



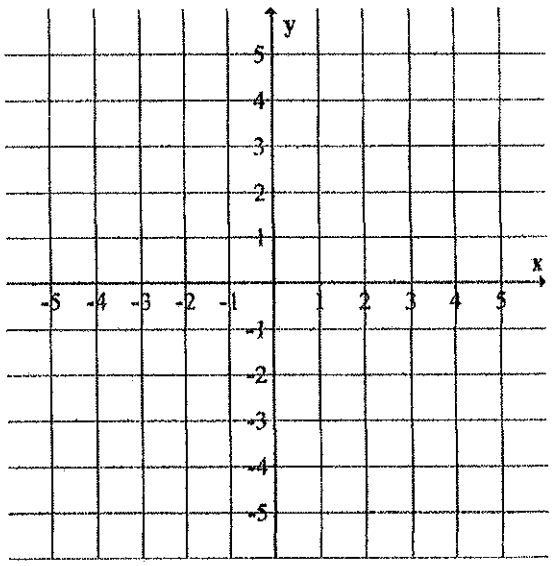
d) _____

3. $y = 1 + \frac{1}{x}$

a) _____

b) _____

c)



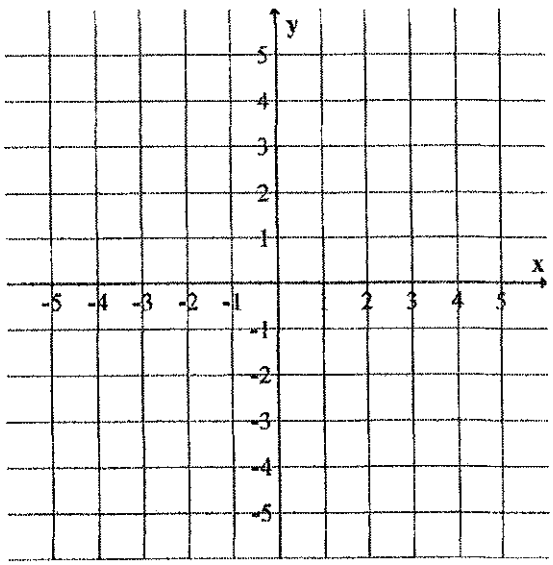
d) _____

4. $y = x^{2/3}$

a) _____

b) _____

c)



d) _____

Determine if the following functions are even, odd, or neither.

5. $y=\frac{1}{x-1}$

6. $y=\frac{1}{x^2-1}$

7. $y=x^4$

8. $y=\sqrt{x^2+2}$

9. $y=\frac{x^3}{x^2-1}$

10. $y=\sqrt[3]{2-x}$

For the following functions find a) $f(g(x))$, b) $g(f(x))$, c) $f(g(0))$, d) $g(f(0))$, e) $g(g(-2))$, and f) $f(f(x))$.

11. $f(x)=x+5,\ g(x)=x^2-3$

a) _____

b) _____

c) _____

d) _____

e) _____

f) _____

12. $f(x)=x+1,\ g(x)=x-1$

a) _____

b) _____

c) _____

d) _____

e) _____

f) _____

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AP Calculus AB – Summer Work – 1.3 Exponential Functions

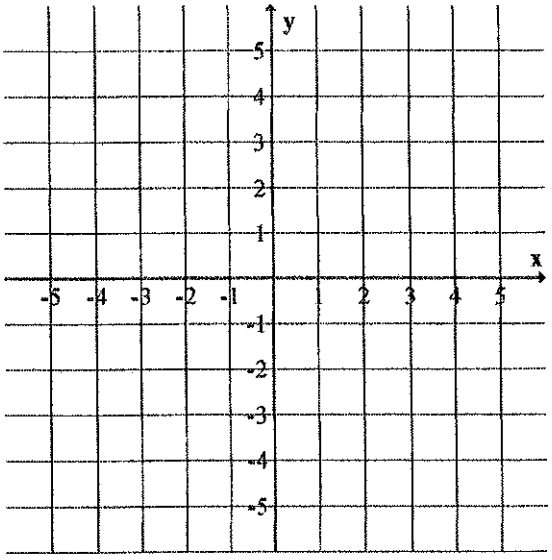
For the following functions find the a) domain, b) range, c) graph, and d) any intercepts.

1. $y = -2^x + 3$

a) _____

b) _____

c)



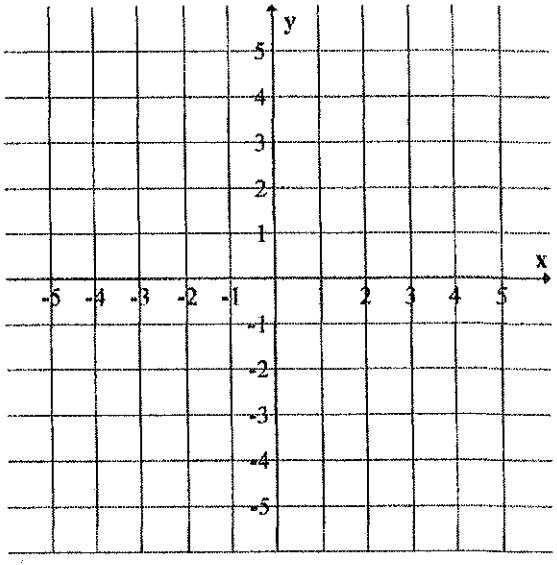
d) _____

2. $y = e^x + 3$

a) _____

b) _____

c)



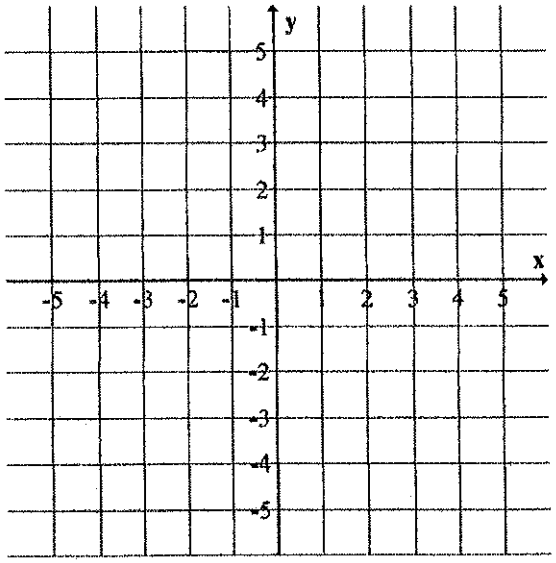
d) _____

3. $y = 3e^{-x} - 2$

a) _____

b) _____

c)



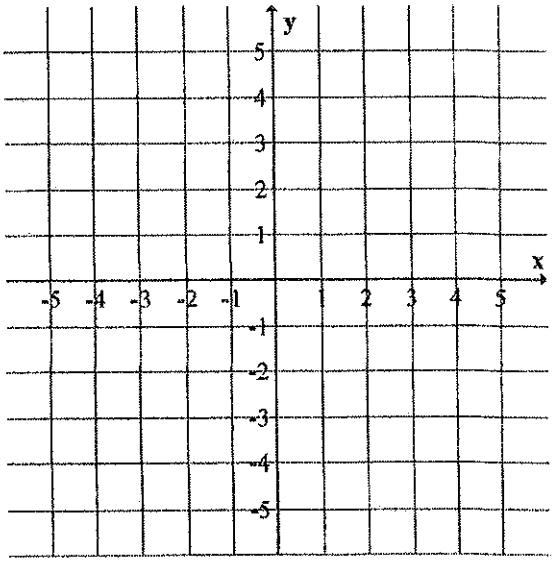
d) _____

4. $y = -2^{-x} - 1$

a) _____

b) _____

c)



d) _____

5. The population of Standardsville is 500,000 and is increasing at the rate of 3.75% each year. Approximately when will the population reach 1 million?

6. The half-life of phosphorus-32 is about 14 days. There are 6.6 grams present initially.
   a) Express the amount of phosphorus-32 remaining as a function of time t.

   b) When will there be 1 gram remaining?

7. Determine how much time is required for an investment to triple if interest is earned at the rate of 4.25% compounded weekly (remember 52 weeks in a year)?

8. Suppose that a colony of bacteria starts with 1 bacterium and doubles in number every half hour. How much bacteria will the colony contain at the end of 24 hours?

Rewrite the following exponential expressions to have the indicated base.

9. 9^{2x}, base 3

10. 16^{3x}, base 2

11. $\left(\frac{1}{27}\right)^{x}$, base 3

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AP Calculus AB – Summer Work – 1.5 Functions and Logarithms

Find f^{-1} and verify that $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$.

1. $f(x) = 2x + 3$

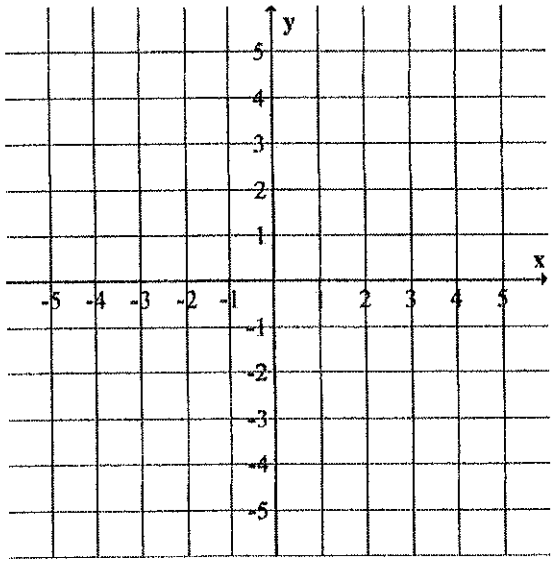
2. $f(x) = x^3 - 1$

3. $f(x) = \dfrac{x+3}{x-2}$

4. $f(x) = x^2 + 2x + 1$

Graph the given function $f(x)$, its inverse $f^{-1}(x)$ and $y = x$ on the same axes in different colors.

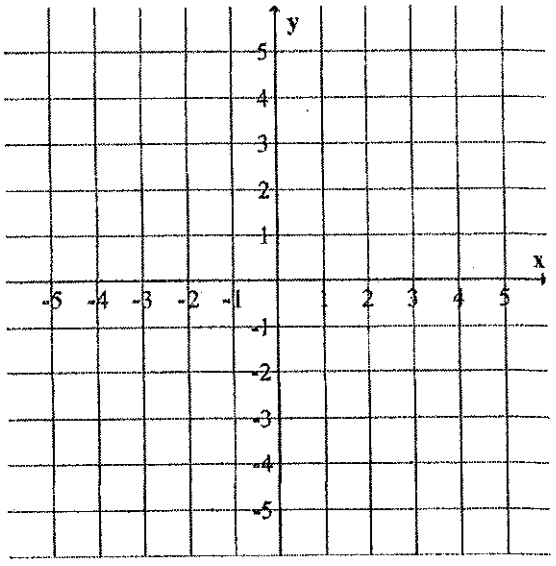
5. $f(x) = e^x$



$f(x) =$

$f^{-1}(x) =$

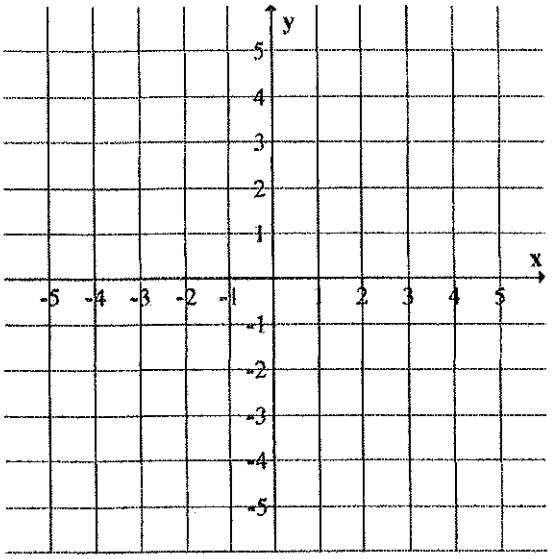
6. $f(x) = 3^x$



$f(x) =$

$f^{-1}(x) =$

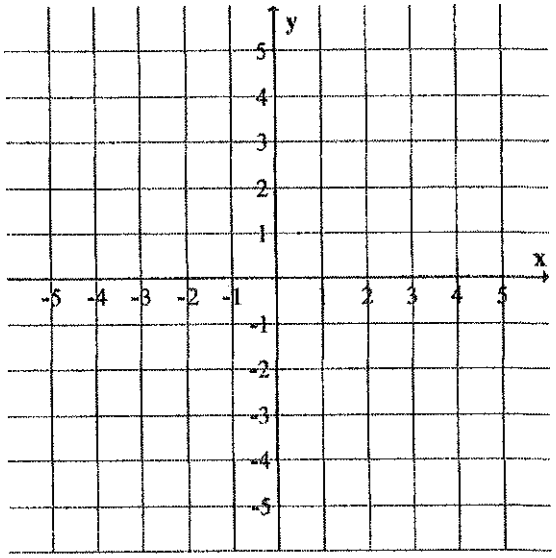
7. $f(x)=\sin^{-1}x$



$f(x)=$

$f^{-1}(x)=$

8. $f(x)=\ln x$



$f(x)=$

$f^{-1}(x)=$

Solve the equation algebraically. Check your solution graphically.

9. $1.045^t=2$

10. $e^{0.05t}=3$

11. $e^x+e^{-x}=3$ (Quad Formula)

Solve for y.

12. $\ln(y-1)-\ln 2=x+\ln x$

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AP Calculus AB – Summer Work – 1.6 Trigonometric Functions

Give the measure of the angle in radians and degrees. Give exact answers whenever possible.

1. $\sin^{-1}(0.5)$

radians: _____

degrees: _____

2. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

radians: _____

degrees: _____

3. $\tan^{-1}(-5)$

radians: _____

degrees: _____

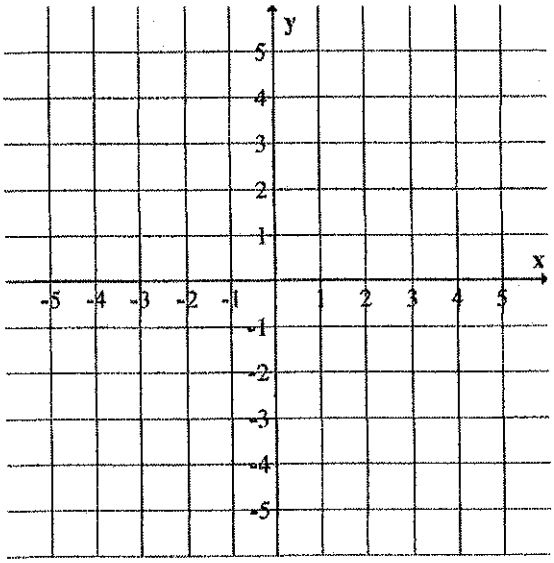
4. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

radians: _____

degrees: _____

Specify the period, the amplitude, and graph the following trigonometric functions.

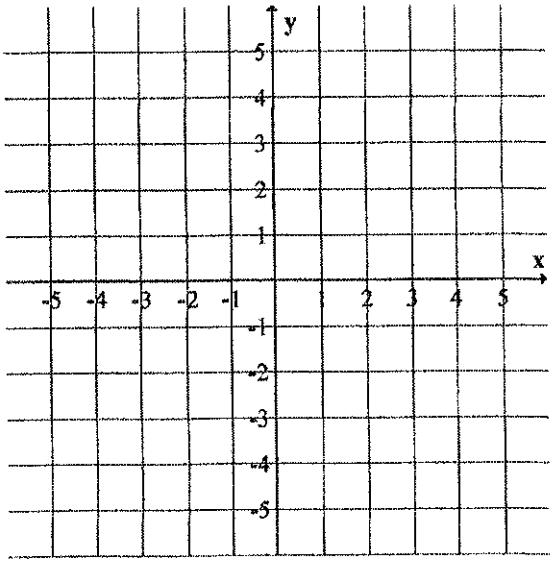
5. $y = 1.5\sin(2x)$



period: _____

amplitude: _____

6. $y = -3\cos(2x)$

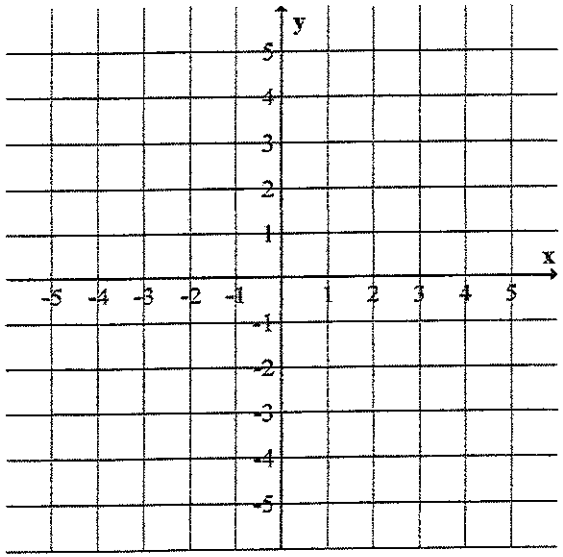


period: _____

amplitude: _____

Specify the period, domain, range, and graph the following trigonometric functions.

7. $y = 3\csc(3x+\pi)-2$

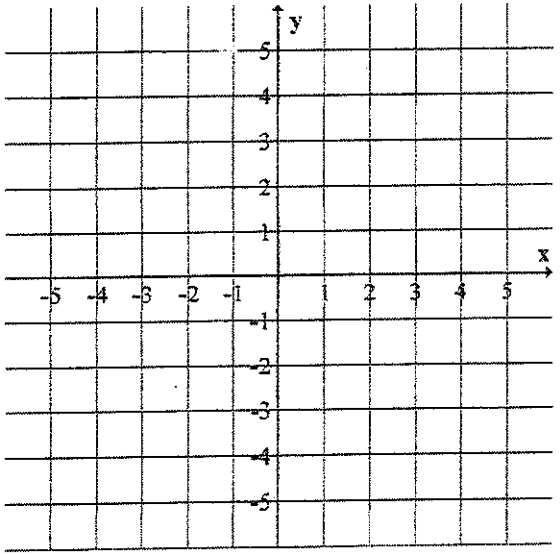


period: _____

domain: _____

range: _____

8. $y = -\tan(3x+\pi)+2$



period: _____

domain: _____

range: _____

Solve the equation in the specified interval.

9. $\cos x = -0.7,\ 2\pi \le x < 4\pi$

10. $\cot x = -1,\ -\infty < x < \infty$

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AP Calculus BC – Summer Work – 2.1 Rates of Change and Limits

Determine the limit by substitution. Support graphically (quick sketch).

1. $\lim_{x\to 1/2} 3x^2(2x-1)$

2. $\lim_{x\to -4} (x+3)^{1998}$

3. $\lim_{x\to 1} (x^3+3x^2-2x-17)$

4. $\lim_{x\to 2} \dfrac{x^2+5x+6}{x+2}$

5. $\lim_{y\to -3} \dfrac{y^2+4y+3}{y^2-3}$

6. $\lim_{x\to \pi/2} \ln(\sin(x))$

Explain why you cannot use substitution to find the limit. Find the limit if it exists.

7. $\lim_{x\to -2} \sqrt{x-2}$

8. $\lim_{x\to 0} \dfrac{1}{x^2}$

9. $\lim_{x\to 0} \dfrac{|x|}{x}$

10. $\lim_{x\to 0} \dfrac{(4+x)^2-16}{x}$

Determine the limit graphically, confirm algebraically.

11. $\lim_{x\to 2}\dfrac{x^2-3x+2}{x^2-4}$

12. $\lim_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

13. $\lim_{x\to 0}\dfrac{\sin(2x)}{x}$

14. $\lim_{x\to 0}\dfrac{\sin x}{2x^2-x}$

Given the graph to the right, find the following limits.

14.

$\lim_{x\to -1^+} f(x) =$

$\lim_{x\to -1^-} f(x) =$

$\lim_{x\to -1} f(x) =$

$\lim_{x\to 0^+} f(x) =$

$\lim_{x\to 0^-} f(x) =$

$\lim_{x\to 0} f(x) =$

$\lim_{x\to 2^-} f(x) =$

$\lim_{x\to 2^+} f(x) =$